The Phantom of Differential Characteristics

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Motivation

For various application scenarios, we often assume the ability of an attacker to control the keys:

- Single-key model
- Open-key model
- Related-key attack
- Weak-key attack
- Known-key attack
Distinguisher +
For various application scenarios, we often assume the ability of an attacker to control the keys: Single-key model, Open-key model, ▶ related-key attack, ▶ weak-key attack, ▶ known-key attack.
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**Distinguisher** + **Attack**

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Motivation

Differential cryptanalysis
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Differential cryptanalysis

- One of the most extensively studied cryptanalytic techniques

\[ E_k(x) \rightarrow y \quad \text{and} \quad E_k(x \oplus \delta) \rightarrow y \oplus \Delta \]
Differential cryptanalysis

- One of the most extensively studied cryptanalytic techniques
- Track probabilistic difference propagation
Differential cryptanalysis

- One of the most extensively studied cryptanalytic techniques
- Track probabilistic difference propagation
- Differential characteristics and differentials
Motivation

Differential cryptanalysis

- One of the most extensively studied cryptanalytic techniques
- Track probabilistic difference propagation
- Differential characteristics and differentials
- Distinguish from random and key recovery
Motivation

An attacker wants to know

- probability of a differential \((\delta, \Delta)\) under a secret key \(k\)
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An attacker wants to know

- probability of a differential $(\delta, \Delta)$ under a secret key $k$
- expected probabilities of a differential $(\delta, \Delta)$ over all master keys

Assumptions

- Markov cipher
- Independently random round keys
- Hypothesis of stochastic equivalence
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- sum on the expected probabilities of all or some characteristics in a differential \((\delta, \Delta)\) over all random round keys
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However, an attacker targets on one secret key.
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However, an attacker targets on one secret key.

- The probability of a differential distinguisher determines the attack complexity
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With the assumptions, it allows to

- estimate the averaged strength of a distinguisher
- provable resistance against differential cryptanalysis
- guideline for designs

However, an attacker targets on one secret key.

- The probability of a differential distinguisher determines the attack complexity
- Differential or impossible differential?
Motivation

Discrepancy observed in previous works:
Motivation

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- ARX ciphers:
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- **ARX ciphers:**
  - Differential cryptanalysis on ARX-based hash functions, see for instance [Leu12]


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  - Rotational cryptanalysis [KNP+15]
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- Plateau characteristics [DR07]
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- Refined differential probability with key being zero [CLN+17]
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- **Plateau characteristics** [DR07]

- **Refined differential probability with key being zero** [CLN+17]

  - . . .

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Motivation

Indepedently random keys
Motivation

Independently random keys

To what extent can we rely on the Assumptions?
Motivation

Enumerate characteristics under the Assumptions:

A characteristic generated under the Assumptions is “almost” impossible in reality.
Enumerate characteristics under the Assumptions:

For a fixed key, $\#$ characteristics = $2^{15}$

Under the Assumptions, $\#$ characteristics = $2^{8} \times 2^{7} \times \cdots \times 2^{7} = 2^{7r+8}$

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- For a fixed key, \# characteristics = $2^{15}$
- Under the Assumptions, \# characteristics $= 2^8 \times 2^7 \times \cdots \times 2^7 = 2^{7r+8}$
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To study differential probability in fixed-key block ciphers and permutations
It is crucial to ask:
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It is crucial to ask:

- **EDP ≠ 0** while **DP = 0** for all keys?
Motivation

To study differential probability in fixed-key block ciphers and permutations
It is crucial to ask:

- $\text{EDP} \neq 0$ while $\text{DP} = 0$ for all keys?
- Differential characteristics enumeration?
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It is crucial to ask:
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- Characteristics-based attacks?
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To study differential probability in fixed-key block ciphers and permutations
It is crucial to ask:

- EDP ≠ 0 while DP = 0 for all keys?
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- Compute DP under any given key?
Motivation

To study differential probability in fixed-key block ciphers and permutations

It is crucial to ask:

- EDP ≠ 0 while DP = 0 for all keys?
- Differential characteristics enumeration?
- Characteristics-based attacks?
- Compute DP under any given key?
- Design better key schedules and/or constants?
Effective Keys and Singular Characteristics

Differential probability is dependent on the key characteristics with zero or nonzero probability. A key is effective for a characteristic if the characteristic is of nonzero probability under the key. If no effective key exists, it is called a singular characteristic.
Effective Keys and Singular Characteristics

- Differential probability is dependent on the key
Effective Keys and Singular Characteristics

- Differential probability is dependent on the key
- Characteristics with zero or nonzero probability
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**Effective keys**

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Effective keys

A key is effective for a characteristic if the characteristic is of nonzero probability under the key.

If no effective key exists, it is called a *singular characteristic*. 
Effective Keys

SPN cipher with keys XORed after the linear layer
Effective Keys

■ SPN cipher with keys XORed after the linear layer
■ Right output and right input of the Sboxes
Effective Keys

- SPN cipher with keys XORed after the linear layer
- Right output and right input of the Sboxes
- Effective key candidates: \( k = Px \oplus y \)
Singular Characteristics

When the difference propagation is legal, the effective key set of a 2-round characteristic is non-empty.

Effective keys derived from two consecutive rounds may not be compatible with the key schedule.
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Singular Characteristics

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Singular Characteristics

\[ \begin{align*}
\alpha_0 & \xrightarrow{S} \beta_0 & \xrightarrow{P} \alpha_1 & \xrightarrow{S} \beta_1 & \xrightarrow{P} \alpha_2 & \xrightarrow{S} \beta_2 & \xrightarrow{P} \alpha_3 & \xrightarrow{S} \beta_3 & \xrightarrow{P} \alpha_4 \\
\end{align*} \]

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Singular Characteristics

\[
\begin{align*}
\alpha_0 & \xrightarrow{S} \beta_0 \xrightarrow{P} \alpha_1 \\
\alpha_1 & \xrightarrow{S} \beta_1 \xrightarrow{P} \alpha_2 \\
\alpha_2 & \xrightarrow{S} \beta_2 \xrightarrow{P} \alpha_3 \\
\alpha_3 & \xrightarrow{S} \beta_3 \xrightarrow{P} \alpha_4
\end{align*}
\]

Key Schedule

Procedure:
Singular Characteristics

\[ \begin{align*}
\alpha_0 & \xrightarrow{S} \beta_0 \xrightarrow{P} \alpha_1 \xrightarrow{S} \beta_1 \xrightarrow{P} \alpha_2 \xrightarrow{S} \beta_2 \xrightarrow{P} \alpha_3 \xrightarrow{S} \beta_3 \xrightarrow{P} \alpha_4
\end{align*} \]

Procedure:
1. Conditions on \( K_i \) to be effective
Singular Characteristics

\[
\begin{align*}
\alpha_0 & \xrightarrow{S} \beta_0 \xrightarrow{P} \alpha_1 \xrightarrow{S} \beta_1 \xrightarrow{P} \alpha_2 \xrightarrow{S} \beta_2 \xrightarrow{P} \alpha_3 \xrightarrow{S} \beta_3 \xrightarrow{P} \alpha_4 \\
K_1 & \\
K_2 & \\
K_3 & \\
k & \rightarrow \text{Key Schedule}
\end{align*}
\]

Procedure:

1. Conditions on \( K_i \) to be effective
2. Conditions based on a specific key schedule
**Singular Characteristics**

\[ \alpha_0 \xrightarrow{S} \beta_0 \xrightarrow{P} \alpha_1 \xrightarrow{S} \beta_1 \xrightarrow{P} \alpha_2 \xrightarrow{S} \beta_2 \xrightarrow{P} \alpha_3 \xrightarrow{S} \beta_3 \xrightarrow{P} \alpha_4 \]

\[ k \rightarrow \text{Key Schedule} \]

**Procedure:**

1. Conditions on \( K_i \) to be effective
2. Conditions based on a specific key schedule
3. Key schedule details
Singular Characteristics

\[ \alpha_0 \xrightarrow{S} \beta_0 \xrightarrow{P} \alpha_1 \xrightarrow{S} \beta_1 \xrightarrow{P} \alpha_2 \xrightarrow{S} \beta_2 \xrightarrow{P} \alpha_3 \xrightarrow{S} \beta_3 \xrightarrow{P} \alpha_4 \]

Key Schedule

 Procedure: 

1. Conditions on \( K_i \) to be effective
2. Conditions based on a specific key schedule
3. Key schedule details
4. Linear equation systems
Singular Characteristics

\[ \alpha_0 \xrightarrow{S} \beta_0 \xrightarrow{P} \alpha_1 \xrightarrow{S} \beta_1 \xrightarrow{P} \alpha_2 \xrightarrow{S} \beta_2 \xrightarrow{P} \alpha_3 \xrightarrow{S} \beta_3 \xrightarrow{P} \alpha_4 \]

Procedure:

1. Conditions on \( K_i \) to be effective
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   - No solution found \( \rightarrow \) singular
### Singular Characteristics

\[
\alpha_0 \xrightarrow{S} \beta_0 \xrightarrow{P} \alpha_1 \xrightarrow{S} \beta_1 \xrightarrow{P} \alpha_2 \xrightarrow{S} \beta_2 \xrightarrow{P} \alpha_3 \xrightarrow{S} \beta_3 \xrightarrow{P} \alpha_4
\]

#### Procedure:

1. Conditions on \( K_i \) to be effective
2. Conditions based on a specific key schedule
3. Key schedule details
4. Linear equation systems
   - No solution found \( \rightarrow \) singular
   - Key candidates found \( \rightarrow \) Further filter by nonlinear constraints
Singular Characteristics in the AES

Find singular characteristics in AES-128:

Picture credit:
TikZ for Cryptographers
Find singular characteristics in AES-128:

- Subspaces of effective keys in every two consecutive rounds

Picture credit: TikZ for Cryptographers
Find singular characteristics in AES-128:

- Subspaces of effective keys in every two consecutive rounds
- Build equation systems with key schedule

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Find singular characteristics in AES-128:

- Subspaces of effective keys in every two consecutive rounds
- Build equation systems with key schedule
- 3 out of 4 columns in AES-128 key schedule are linear relations
Find singular characteristics in AES-128:

- Subspaces of effective keys in every two consecutive rounds
- Build equation systems with key schedule
- 3 out of 4 columns in AES-128 key schedule are linear relations
- Simplify and solve the equation system
Singular Characteristics in the AES

Examples of 5-round singular characteristics can be found in the AES-128.

\[
\begin{pmatrix}
1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0
\end{pmatrix}
\xrightarrow{S}
\begin{pmatrix}
1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0
\end{pmatrix}
\xrightarrow{P}
\begin{pmatrix}
2 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 \\
3 & 0 & 0 & 0
\end{pmatrix}
\xrightarrow{S}
\begin{pmatrix}
3 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 \\
2 & 0 & 0 & 0
\end{pmatrix}
\]

\[
\begin{pmatrix}
6 & 2 & 1 & 3 \\
3 & 2 & 3 & 2 \\
3 & 6 & 2 & 1 \\
5 & 4 & 1 & 1
\end{pmatrix}
\xrightarrow{P}
\begin{pmatrix}
24 & 27 & 39 & 9d \\
45 & 36 & 36 & 27 \\
36 & f1 & 2e & 2d \\
39 & 2d & 1f & 3a
\end{pmatrix}
\xrightarrow{S}
\begin{pmatrix}
6 & 0 & 0 & 0 \\
0 & 5 & 0 & 0 \\
0 & 0 & 5 & 0 \\
0 & 0 & 0 & 36
\end{pmatrix}
\]

\[
\begin{pmatrix}
e & 0 & 0 & 0 \\
0 & 9 & 0 & 0 \\
0 & d & 0 & 0 \\
0 & 0 & b & 0
\end{pmatrix}
\xrightarrow{S}
\begin{pmatrix}
1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0
\end{pmatrix}
\xrightarrow{P}
\begin{pmatrix}
1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0
\end{pmatrix}
\xrightarrow{S}
\begin{pmatrix}
1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0
\end{pmatrix}.
\]
Singular Characteristics in the AES

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\[
\begin{pmatrix}
* & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
\end{pmatrix}
\xrightarrow{S}
\begin{pmatrix}
* & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
\end{pmatrix}
\xrightarrow{P}
\begin{pmatrix}
* & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
\end{pmatrix}
\xrightarrow{S}
\begin{pmatrix}
* & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
\end{pmatrix}
\xrightarrow{P}
\begin{pmatrix}
* & * & * & * \\
* & * & * & * \\
* & * & * & * \\
* & * & * & * \\
\end{pmatrix}
\xrightarrow{S}
\begin{pmatrix}
* & * & * & * \\
* & * & * & * \\
* & * & * & * \\
* & * & * & * \\
\end{pmatrix}
\xrightarrow{P}
\begin{pmatrix}
* & * & * & * \\
* & * & * & * \\
* & * & * & * \\
* & * & * & * \\
\end{pmatrix}
\xrightarrow{S}
\begin{pmatrix}
* & * & * & * \\
* & * & * & * \\
* & * & * & * \\
* & * & * & * \\
\end{pmatrix}
\xrightarrow{P}
\begin{pmatrix}
* & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
\end{pmatrix}
\xrightarrow{S}
\begin{pmatrix}
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
\end{pmatrix}
\]

MITM attack
Density of singular characteristics:

More than 98.47% of all the characteristics are singular for the remaining characteristics, we consider the nonlinear constraints from the key schedule and get their effective keys ▶ some of them may also be singular ▶ the number of effective keys is around $2^{7}$ to $2^{10}$. 
Singular Characteristics in the AES

Density of singular characteristics:

\[
\begin{pmatrix}
* & 0 & 0 & 0 \\
* & 0 & 0 & 0 \\
* & 0 & 0 & 0 \\
* & 0 & 0 & 0 \\
\end{pmatrix}
\rightarrow
\begin{pmatrix}
* & 0 & 0 & 0 \\
* & 0 & 0 & 0 \\
* & 0 & 0 & 0 \\
* & 0 & 0 & 0 \\
\end{pmatrix}
\rightarrow
\begin{pmatrix}
**** \\
**** \\
**** \\
**** \\
\end{pmatrix}
\rightarrow
\begin{pmatrix}
**** \\
**** \\
**** \\
**** \\
\end{pmatrix}
\rightarrow
\begin{pmatrix}
* & 0 & 0 & 0 \\
* & 0 & 0 & 0 \\
0 & 0 & * & 0 \\
0 & 0 & 0 & 0 \\
\end{pmatrix}
\rightarrow
\begin{pmatrix}
* & 0 & 0 & 0 \\
* & 0 & 0 & 0 \\
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0 & 0 & 0 & 0 \\
\end{pmatrix}
\]
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\[
\begin{pmatrix}
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0 & 0 & 0 & 0 \\
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0 & 0 & 0 & 0
\end{pmatrix}
\xrightarrow{S} \begin{pmatrix}
* & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0
\end{pmatrix}
\xrightarrow{P} \begin{pmatrix}
* & * & * & * \\
* & * & * & * \\
* & * & * & * \\
* & * & * & *
\end{pmatrix}
\xrightarrow{S} \begin{pmatrix}
* & * & * & * \\
* & * & * & * \\
* & * & * & * \\
* & * & * & *
\end{pmatrix}
\xrightarrow{P} \begin{pmatrix}
* & 0 & 0 & 0 \\
0 & * & 0 & 0 \\
0 & 0 & * & 0 \\
0 & 0 & 0 & 0
\end{pmatrix}
\xrightarrow{S} \begin{pmatrix}
* & 0 & 0 & 0 \\
0 & * & 0 & 0 \\
0 & 0 & * & 0 \\
0 & 0 & 0 & *
\end{pmatrix}
\]

Enumerate all characteristics given a 3-round differential.
Density of singular characteristics:

\[
\begin{pmatrix}
* & 0 & 0 & 0 \\
* & 0 & 0 & 0 \\
* & 0 & 0 & 0 \\
* & 0 & 0 & 0 \\
\end{pmatrix} 
\rightarrow 
\begin{pmatrix}
* & 0 & 0 & 0 \\
* & 0 & 0 & 0 \\
* & 0 & 0 & 0 \\
* & 0 & 0 & 0 \\
\end{pmatrix} 
\rightarrow 
\begin{pmatrix}
* & * & * & * \\
* & * & * & * \\
* & * & * & * \\
* & * & * & * \\
\end{pmatrix} 
\rightarrow 
\begin{pmatrix}
* & * & * & * \\
* & * & * & * \\
* & * & * & * \\
* & * & * & * \\
\end{pmatrix} 
\rightarrow 
\begin{pmatrix}
* & 0 & 0 & 0 \\
0 & * & 0 & 0 \\
0 & 0 & * & 0 \\
0 & 0 & 0 & * \\
\end{pmatrix} 
\rightarrow 
\begin{pmatrix}
* & 0 & 0 & 0 \\
0 & * & 0 & 0 \\
0 & 0 & * & 0 \\
0 & 0 & 0 & * \\
\end{pmatrix}
\]

- Enumerate all characteristics given a 3-round differential
- More than 98.47% of all the characteristics are singular
Singular Characteristics in the AES

Density of singular characteristics:

\[
\begin{pmatrix}
* & 0 & 0 & 0 \\
* & 0 & 0 & 0 \\
* & 0 & 0 & 0 \\
* & 0 & 0 & 0 \\
\end{pmatrix}
\xrightarrow{S} 
\begin{pmatrix}
* & 0 & 0 & 0 \\
* & 0 & 0 & 0 \\
* & 0 & 0 & 0 \\
* & 0 & 0 & 0 \\
\end{pmatrix}
\xrightarrow{P} 
\begin{pmatrix}
* & * & * & * \\
* & * & * & * \\
* & * & * & * \\
* & * & * & * \\
\end{pmatrix}
\xrightarrow{S} 
\begin{pmatrix}
* & * & * & * \\
* & * & * & * \\
* & * & * & * \\
* & * & * & * \\
\end{pmatrix}
\xrightarrow{P} 
\begin{pmatrix}
* & 0 & 0 & 0 \\
0 & * & 0 & 0 \\
0 & 0 & * & 0 \\
0 & 0 & 0 & * \\
\end{pmatrix}
\xrightarrow{S} 
\begin{pmatrix}
* & 0 & 0 & 0 \\
0 & * & 0 & 0 \\
0 & 0 & * & 0 \\
0 & 0 & 0 & * \\
\end{pmatrix}
\]

- Enumerate all characteristics given a 3-round differential
- More than 98.47% of all the characteristics are singular
- For the remaining characteristics, we consider the nonlinear constraints from the key schedule and get their effective keys
Density of singular characteristics:

\[
\begin{pmatrix}
* & 0 & 0 & 0 \\
* & 0 & 0 & 0 \\
* & 0 & 0 & 0 \\
* & 0 & 0 & 0 \\
\end{pmatrix}
\rightarrow
\begin{pmatrix}
* & 0 & 0 & 0 \\
* & 0 & 0 & 0 \\
* & 0 & 0 & 0 \\
* & 0 & 0 & 0 \\
\end{pmatrix}
\rightarrow
\begin{pmatrix}
* & * & * & * \\
* & * & * & * \\
* & * & * & * \\
* & * & * & * \\
\end{pmatrix}
\rightarrow
\begin{pmatrix}
* & 0 & 0 & 0 \\
0 & * & 0 & 0 \\
0 & 0 & * & 0 \\
0 & 0 & 0 & * \\
\end{pmatrix}
\rightarrow
\begin{pmatrix}
* & 0 & 0 & 0 \\
0 & * & 0 & 0 \\
0 & 0 & * & 0 \\
0 & 0 & 0 & * \\
\end{pmatrix}
\]

- Enumerate all characteristics given a 3-round differential
- More than 98.47% of all the characteristics are singular
- For the remaining characteristics, we consider the nonlinear constraints from the key schedule and get their effective keys
  - some of them may also be singular
  - the number of effective keys is around $2^7$ to $2^{10}$
Different key schedules affect the singularity of a characteristic.
Different key schedules affect the singularity of a characteristic

- Encrypt a pair of plaintexts under some key with AES-128, track the characteristic
Singular Characteristics in the AES

Different key schedules affect the singularity of a characteristic

- Encrypt a pair of plaintexts under some key with AES-128, track the characteristic
- Change the key schedule into AES-192
Different key schedules affect the singularity of a characteristic

- Encrypt a pair of plaintexts under some key with AES-128, track the characteristic
- Change the key schedule into AES-192
- A valid characteristic in AES-128 is highly probable to be singular in AES-192
Singular Characteristics in the AES

- Different key schedules affect the singularity of a characteristic
  - Encrypt a pair of plaintexts under some key with AES-128, track the characteristic
  - Change the key schedule into AES-192
  - A valid characteristic in AES-128 is highly probable to be singular in AES-192
- Differential enumeration + key schedule constraints
Singular Characteristics in the AES

Different key schedules affect the singularity of a characteristic

- Encrypt a pair of plaintexts under some key with AES-128, track the characteristic
- Change the key schedule into AES-192
- A valid characteristic in AES-128 is highly probable to be singular in AES-192

- Differential enumeration + key schedule constraints
- Extension to AES-like, Feistel-SP, Feistel
Singular Characteristics in Prince

A 3-round singular characteristic with $\text{EDP} = \frac{-35}{18}$
Singular Characteristics in Prince

\[
\begin{pmatrix}
8 & 0 & 4 & 0 \\
0 & 0 & 0 & 0 \\
4 & 0 & 8 & 0 \\
0 & 0 & 0 & 0 \\
\end{pmatrix}
\stackrel{S}{\rightarrow}
\begin{pmatrix}
8 & 0 & 4 & 0 \\
0 & 0 & 0 & 0 \\
8 & 0 & 4 & 0 \\
0 & 0 & 0 & 0 \\
\end{pmatrix}
\stackrel{M'}{\rightarrow}
\begin{pmatrix}
8 & 0 & 4 & 0 \\
0 & 0 & 0 & 0 \\
8 & 0 & 4 & 0 \\
0 & 0 & 0 & 0 \\
\end{pmatrix}
\stackrel{SR}{\rightarrow}
\begin{pmatrix}
8 & 0 & 4 & 0 \\
0 & 0 & 0 & 0 \\
4 & 0 & 8 & 0 \\
0 & 0 & 0 & 0 \\
\end{pmatrix}
\stackrel{S}{\rightarrow}
\begin{pmatrix}
8 & 0 & 5 & 0 \\
0 & 0 & 0 & 0 \\
8 & 0 & 5 & 0 \\
0 & 0 & 0 & 0 \\
\end{pmatrix}
\stackrel{M'}{\rightarrow}
\begin{pmatrix}
8 & 0 & 5 & 0 \\
0 & 0 & 0 & 0 \\
8 & 0 & 5 & 0 \\
0 & 0 & 0 & 0 \\
\end{pmatrix}
\stackrel{SR}{\rightarrow}
\begin{pmatrix}
8 & 0 & 5 & 0 \\
0 & 0 & 0 & 0 \\
5 & 0 & 8 & 0 \\
0 & 0 & 0 & 0 \\
\end{pmatrix}
\stackrel{S}{\rightarrow}
\begin{pmatrix}
2 & 0 & 5 & 0 \\
0 & 0 & 0 & 0 \\
2 & 0 & 5 & 0 \\
0 & 0 & 0 & 0 \\
\end{pmatrix}
\]
Singular Characteristics in Prince

A 3-round singular characteristic with EDP $= 2^{-35}$
If no effective key in common → singular cluster.

Differentials/truncated differentials/multiple differentials
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Differentials/truncated differentials/multiple differentials

Singular Cluster

\[ \alpha_0 \xrightarrow{S} \beta_0 \xrightarrow{P} \alpha_1 \xrightarrow{S} \beta_1 \xrightarrow{P} \alpha_2 \xrightarrow{S} \beta_2 \xrightarrow{P} \alpha_3 \xrightarrow{S} \beta_3 \xrightarrow{P} \alpha_4 \]

\[ \alpha'_0 \xrightarrow{S} \beta'_0 \xrightarrow{P} \alpha'_1 \xrightarrow{S} \beta'_1 \xrightarrow{P} \alpha'_2 \xrightarrow{S} \beta'_2 \xrightarrow{P} \alpha'_3 \xrightarrow{S} \beta'_3 \xrightarrow{P} \alpha'_4 \]
Singular Cluster

\[ \alpha_0 \xrightarrow{S} \beta_0 \xrightarrow{P} \alpha_1 \xrightarrow{S} \beta_1 \xrightarrow{P} \alpha_2 \xrightarrow{S} \beta_2 \xrightarrow{P} \alpha_3 \xrightarrow{S} \beta_3 \xrightarrow{P} \alpha_4 \]

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Singular Cluster

If no effective key in common → *singular cluster.*
Differentials/truncated differentials/multiple differentials
Observation: If a differential contains only singular characteristics, it is an impossible differential.
Further Applications

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- Improve distinguishers?
Consider a 5-round differential $\mathcal{D}$ of the AES with active pattern 1-4-16-4-1. The effective keys of each characteristic can be precomputed.

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  - Information leaked about the secret key
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Summary

- Differential cryptanalysis in fixed-key block ciphers and permutations
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- Effective keys and singular characteristics are proposed based on fixed-key DP

Pay extra attention to characteristics generated from enumeration techniques when they are applied in attacks.

New approach towards improved distinguisher or key recovery technique.

Thank you for your attention!
Summary

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